Accelerated Pre-Calculus Honors Summer Assignment 2018

Welcome!

This packet includes a sampling of problems that students entering Accelerated Pre-Calculus Honors should be able to answer.

Pre-Calculus course is the gateway to any future mathematics course and many science courses. Success will be attained through a positive attitude, hard work, and perseverance, especially when topics seem extremely difficult.

The work will tap into your prior knowledge and review past content, concepts, and skills. Our expectation is that you arrive on the first day of school able to demonstrate mastery of the materials in this packet. In order to achieve this, please allow yourself plenty of time to work on the problems and work each problem to completion.

Part I – Khan Academy – YOU MUST LOG IN TO HAVE KHAN ACADEMY SAVE YOUR WORK so that YOU CAN RECEIVE CREDIT for YOUR WORK! (Next page has more details on Khan Academy assignment.)

How to Log Into Khan Academy to complete your summer work:

- If you do not have an account yet:

1. Go to khanacademy.org
2. Click on Start Learning Now
3. Click on Sign in with Gmail
4. Create an Account
5. Type in the Search Engine Bar The topic you would like to complete or click on the link provided

- If you have an account already

1. Go to khanacademy.org
2. Click on the log in button on the top right hand side of the screen
3. Log in with your email and password
4. Type in the Search Engine Bar the topic you would like to complete or click on the link provided

Part II – Please complete the rest of this packet and show & save your work for all problems.

Please let me know if you have any questions at sduzyol@fultonscienceacademy.org.

Part I – Khan Academy – YOU MUST LOG IN TO HAVE KHAN ACADEMY SAVE YOUR WORK so that YOU CAN RECEIVE CREDIT for YOUR WORK!

Khan Academy Summer Work Topics for Pre-Calculus:

a. Factor Perfect Squares http://tinyurl.com/zv9y8lb

b. Factoring Quadratics ( ) http://tinyurl.com/pzjz9eu

c. Factor Polynomials: Quadratics Advanced http://tinyurl.com/qxg9ccq

d. Find Zeros of Polynomials http://tinyurl.com/jfoafpn

e. End Behavior of Polynomials: Even / Odd Function http://tinyurl.com/z2v2rlo

f. Distance Formula http://tinyurl.com/pmebbx3

g. Special Right Triangles http://tinyurl.com/mncy46w

h. Completing the Square (intermediate) http://tinyurl.com/nlfwhzb

i. Quadratic Formula: Solving Quadratic Equations http://tinyurl.com/pmze7xj

j. Midpoint Formula http://tinyurl.com/mdmjc6h

* If you struggle on any of the problems, please reference the videos for each topic and/or look for hints found on the right hand side of your screen when attempting a problem. Please note you do not have to complete all problems in one sitting. Khan Academy will remember where you left off. **Be sure to log-in every time you do additional work!!**

Part II

Solving Absolute Value Equations

Example Solve $|2x - 3| = 17$. Check your solutions.

Case 1

$$a = b$$
$$2x - 3 = 17$$
$$2x - 3 + 3 = 17 + 3$$
$$2x = 20$$
$$x = 10$$

CHECK
$$|2x - 3| = 17$$
$$|2(10) - 3| = 17$$
$$|20 - 3| = 17$$
$$|17| = 17$$
$$17 = 17 \checkmark$$

Case 2

$$a = -b$$
$$2x - 3 = -17$$
$$2x - 3 + 3 = -17 + 3$$
$$2x = -14$$
$$x = -7$$

CHECK
$$|2(-7) - 3| = 17$$
$$|-14 - 3| = 17$$
$$|-17| = 17$$
$$17 = 17 \checkmark$$

There are two solutions, 10 and -7.

1. Solve $|5x + 9| + 16 = 22$. Check your solutions.
2. Solve $5x + 24 = |8 - 3x|$. Check your solutions.

Relations and Functions

Example Given the function $f(x) = x^2 + 2x$, find each value.

a. $f(3)$

$f(x) = x^2 + 2x$ Original function

$f(3) = 3^2 + 2(3)$ Substitute.

$= 15$ Simplify.

b. $f(5a)$

$f(x) = x^2 + 2x$ Original function

$f(5a) = (5a)^2 + 2(5a)$ Substitute.

$= 25a^2 + 10a$ Simplify.

3. Given the function $f(x) = -3x^2 - 7x + 23$, find:
   a. $f(-4)$
   b. $f(n + 5)$

Linear Equations

Slope:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Standard Form:

$Ax + By = C$ (remember, no fractions or decimals)

Point-Slope Form:

$y - y_1 = m(x - x_1)$, where (x_1, y_1) are the coordinates of a point on the line and m is the slope of the line.

Slope-Intercept Form:

$y = mx + b$, where m is the slope and b is the y-intercept

Example 1 **Write an equation in slope-intercept form for the line that has slope −2 and passes through the point (3, 7).**

Substitute for m, x, and y in the slope-intercept form.

$y = mx + b$	Slope-intercept form
$7 = (-2)(3) + b$	$(x, y) = (3, 7)$, $m = -2$
$7 = -6 + b$	Simplify.
$13 = b$	Add 6 to both sides

The y-intercept is 13. The equation in slope-intercept form is $y = -2x + 13$.

Example 2 **Write an equation in slope-intercept form for the line that has slope $\frac{1}{3}$ and x-intercept 5.**

$y = mx + b$	Slope-intercept form
$0 = \left(\frac{1}{3}\right)(5) + b$	$(x, y) = (5, 0)$, $m = \frac{1}{3}$
$0 = \frac{5}{3} + b$	Simplify.
$-\frac{5}{3} = b$	Subtract $\frac{5}{3}$ from both sides.

The y-intercept is $-\frac{5}{3}$. The slope-intercept form is $y = \frac{1}{3}x - \frac{5}{3}$.

4. Find the slope of the line containing the points (-5,4) and (6,-9).

5. Determine the slope of the line $7x - 4y = 12$

6. Find the standard form of a line that contains the point (5,-7) and has a slope $m = -\frac{13}{5}$.

7. Find the slope-intercept form of a line passing through the points $(-4,1) and$ $(5,2)$

Parallel and Perpendicular Lines

Example 1 **Write an equation of the line that passes through (8, 2) and is perpendicular to the line whose equation is $y = -\frac{1}{2}x + 3$.**

The slope of the given line is $-\frac{1}{2}$. Since the slopes of perpendicular lines are negative reciprocals, the slope of the perpendicular line is 2.

Use the slope and the given point to write the equation.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - 2 = 2(x - 8)$	$(x_1, y_1) = (8, 2)$, $m = 2$
$y - 2 = 2x - 16$	Distributive Prop.
$y = 2x - 14$	Add 2 to each side.

An equation of the line is $y = 2x - 14$.

Example 2 **Write an equation of the line that passes through (−1, 5) and is parallel to the graph of $y = 3x + 1$.**

The slope of the given line is 3. Since the slopes of parallel lines are equal, the slope of the parallel line is also 3.

Use the slope and the given point to write the equation.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - 5 = 3(x - (-1))$	$(x_1, y_1) = (-1, 5)$, $m = 3$
$y - 5 = 3x + 3$	Distributive Prop.
$y = 3x + 8$	Add 5 to each side.

An equation of the line is $y = 3x + 8$.

8. Write the slope-intercept form of the equation of a line passing through the point (2,1) and perpendicular to the line $4x - 2y = 3$.

9. Write the slope-intercept form of the equation of a line passing through the point (1, -1), parallel to the line passing through the points (4, 1) and (2, -3).

Simplifying Radicals

Example 1 **Simplify $\sqrt{180}$.**

$\sqrt{180} = \sqrt{2 \cdot 2 \cdot 3 \cdot 3 \cdot 5}$	Prime factorization of 180
$= \sqrt{2^2} \cdot \sqrt{3^2} \cdot \sqrt{5}$	Product Property of Square Roots
$= 2 \cdot 3 \cdot \sqrt{5}$	Simplify.
$= 6\sqrt{5}$	Simplify.

Example 2 **Simplify $\sqrt{120a^2 \cdot b^5 \cdot c^4}$.**

$$\sqrt{120a^2 \cdot b^5 \cdot c^4}$$
$$= \sqrt{2^3 \cdot 3 \cdot 5 \cdot a^2 \cdot b^5 \cdot c^4}$$
$$= \sqrt{2^2} \cdot \sqrt{2} \cdot \sqrt{3} \cdot \sqrt{5} \cdot \sqrt{a^2} \cdot \sqrt{b^4 \cdot b} \cdot \sqrt{c^4}$$
$$= 2 \cdot \sqrt{2} \cdot \sqrt{3} \cdot \sqrt{5} \cdot |a| \cdot b^2 \cdot \sqrt{b} \cdot c^2$$
$$= 2|a|b^2c^2\sqrt{30b}$$

10. $\sqrt{162}$

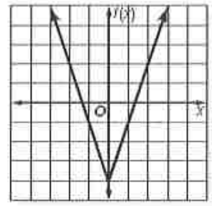
11. $\sqrt{150a^2b^2c}$

Special Functions

Example 1 **Graph $f(x) = 3|x| - 4$.**

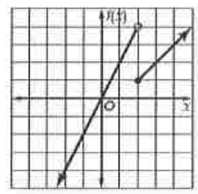
Find several ordered pairs. Graph the points and connect them. You would expect the graph to look similar to its parent function, $f(x) = |x|$.

x	$3|x| - 4$
0	−4
1	−1
2	2
−1	−1
−2	2



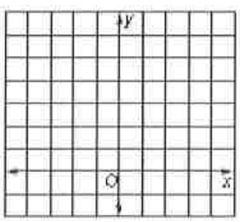
Example 2 **Graph $f(x) = \begin{cases} 2x \text{ if } x < 2 \\ x - 1 \text{ if } x \geq 2. \end{cases}$**

First, graph the linear function $f(x) = 2x$ for $x < 2$. Since 2 does not satisfy this inequality, stop with a circle at (2, 4). Next, graph the linear function $f(x) = x - 1$ for $x \geq 2$. Since 2 does satisfy this inequality, begin with a dot at (2, 1).

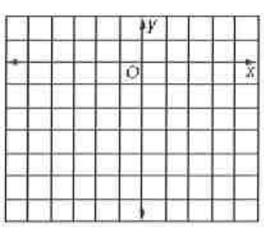


Graph the functions:

12. $h(x) = |2x + 1|$



13. $h(x) = \begin{cases} \frac{x}{3} \text{ if } x \leq 0 \\ 2x - 6 \text{ if } 0 < x < 2 \\ 1 \text{ if } x \geq 2 \end{cases}$



Systems of Equations

Example **Use substitution to solve the system of equations.** $2x - y = 9$, $x + 3y = -6$

Solve the first equation for y in terms of x.

$2x - y = 9$	First equation
$-y = -2x + 9$	Subtract 2x from both sides.
$y = 2x - 9$	Multiply both sides by −1.

Substitute the expression $2x - 9$ for y into the second equation and solve for x.

$x + 3y = -6$	Second equation
$x + 3(2x - 9) = -6$	Substitute 2x − 9 for y.
$x + 6x - 27 = -6$	Distributive Property
$7x - 27 = -6$	Simplify.
$7x = 21$	Add 27 to each side.
$x = 3$	Divide each side by 7.

Now, substitute the value 3 for x in either original equation and solve for y.

$2x - y = 9$	First equation
$2(3) - y = 9$	Replace x with 3.
$6 - y = 9$	Simplify.
$-y = 3$	Subtract 6 from each side.
$y = -3$	Multiply each side by −1.

The solution of the system is $(3, -3)$.

Example 2 **Use the elimination method to solve the system of equations.**

$$3x - 2y = 4$$
$$5x + 3y = -25$$

Multiply the first equation by 3 and the second equation by 2. Then add the equations to eliminate the y variable.

$3x - 2y = 4$	Multiply by 3.	$9x - 6y = 12$
$5x + 3y = -25$	Multiply by 2.	$10x + 6y = -50$
		$19x = -38$
		$x = -2$

Replace x with -2 and solve for y.

$$3x - 2y = 4$$
$$3(-2) - 2y = 4$$
$$-6 - 2y = 4$$
$$-2y = 10$$
$$y = -5$$

The solution is $(-2, -5)$.

14. Solve the system of equations: $\begin{cases} 2x - y = 7 \\ 3x + y = 8 \end{cases}$

15. Solve the system of equations: $\begin{cases} 5x + 2y = -8 \\ 4x + 3y = 2 \end{cases}$

16. Solve the system of equations: $\begin{cases} 2y = 3x - 7 \\ 4x = 3y + 10 \end{cases}$

Factoring

Example 1 **Factor each trinomial.**

a. $x^2 + 7x + 10$

In this trinomial, $b = 7$ and $c = 10$.

Factors of 10	Sum of Factors
1, 10	11
2, 5	7

Since $2 + 5 = 7$ and $2 \cdot 5 = 10$, let $m = 2$ and $n = 5$.

$x^2 + 7x + 10 = (x + 5)(x + 2)$

Example 2 **Factor $6ax + 3ay + 2bx + by$ by grouping.**

$6ax + 3ay + 2bx + by$
$= (6ax + 3ay) + (2bx + by)$
$= 3a(2x + y) + b(2x + y)$
$= (3a + b)(2x + y)$

Check using the FOIL method.

$(3a + b)(2x + y)$
$= 3a(2x) + (3a)(y) + (b)(2x) + (b)(y)$
$= 6ax + 3ay + 2bx + by$ ✓

Example 1 **Factor $2x^2 + 15x + 18$.**

In this example, $a = 2$, $b = 15$, and $c = 18$. You need to find two numbers whose sum is 15 and whose product is $2 \cdot 18$ or 36. Make a list of the factors of 36 and look for the pair of factors whose sum is 15.

Factors of 36	Sum of Factors
1, 36	37
2, 18	20
3, 12	15

Use the pattern $ax^2 + mx + nx + c$, with $a = 2$, $m = 3$, $n = 12$, and $c = 18$.

$2x^2 + 15x + 18 = 2x^2 + 3x + 12x + 18$
$= (2x^2 + 3x) + (12x + 18)$
$= x(2x + 3) + 6(2x + 3)$
$= (x + 6)(2x + 3)$

Therefore, $2x^2 + 15x + 18 = (x + 6)(2x + 3)$.

Example 2 **Factor each polynomial.**

a. $50a^2 - 72$

$50a^2 - 72$
$= 2(25a^2 - 36)$ Find the GCF.
$= 2[(5a)^2 - 6^2)]$ $25a^2 = 5a \cdot 5a$ and $36 = 6 \cdot 6$
$= 2(5a + 6)(5a - 6)$ Factor the difference of squares.

b. $4x^4 + 8x^3 - 4x^2 - 8x$

$4x^4 + 8x^3 - 4x^2 - 8x$ Original polynomial
$= 4x(x^3 + 2x^2 - x - 2)$ Find the GCF.
$= 4x[(x^3 + 2x^2) - (x + 2)]$ Group terms.
$= 4x[x^2(x + 2) - 1(x + 2)]$ Find the GCF.
$= 4x[(x^2 - 1)(x + 2)]$ Factor by grouping.
$= 4x[(x - 1)(x + 1)(x + 2)]$ Factor the difference of squares.

17. $2x^2 - 3x - 2$

18. $16x^2 - 8x + 1$

19. $6x^2 + 5x - 6$

20. $8x^2 - 4x - 24$

21. $a^2 - 4ab + 4b^2$

22. $36x^2 - 100y^2$

23. $72x^2 - 50$

24. $8x^3 - 128x$

Solving Equations by Factoring

Example **Solve each equation by factoring.**

a. $3x^2 = 15x$

$3x^2 = 15x$ Original equation

$3x^2 - 15x = 0$ Subtract $15x$ from both sides.

$3x(x - 5) = 0$ Factor the binomial.

$3x = 0$ or $x - 5 = 0$ Zero Product Property

$x = 0$ or $x = 5$ Solve each equation.

The solution set is $\{0, 5\}$.

b. $4x^2 - 5x = 21$

$4x^2 - 5x = 21$ Original equation

$4x^2 - 5x - 21 = 0$ Subtract 21 from both sides.

$(4x + 7)(x - 3) = 0$ Factor the trinomial.

$4x + 7 = 0$ or $x - 3 = 0$ Zero Product Property

$x = -\frac{7}{4}$ or $x = 3$ Solve each equation.

The solution set is $\left\{-\frac{7}{4}, 3\right\}$.

Solve the equations by factoring:

25. $6x^2 - 2x = 0$

26. $x^2 + x - 30 = 0$

27. $6x^2 - 5x - 4 = 0$

28. $12x^2 = -18x - 6$

Simplifying Rational Expressions

Example 2 **Simplify $\frac{3x - 9}{x^2 - 5x + 6}$. State the excluded values of x.**

$\frac{3x - 9}{x^2 - 5x + 6} = \frac{3(x - 3)}{(x - 2)(x - 3)}$ Factor.

$= \frac{3(x - 3)^1}{(x - 2)(x - 3)_1}$ Divide by the GCF, $x - 3$.

$= \frac{3}{x - 2}$ Simplify.

Exclude the values for which $x^2 - 5x + 6 = 0$.

$x^2 - 5x + 6 = 0$

$(x - 2)(x - 3) = 0$

$x = 2$ or $x = 3$

Therefore, $x \neq 2$ and $x \neq 3$.

Simplify the rational expression. State the excluded value.

29. $\frac{x^2 - 4}{x^2 + 6x + 8}$

30. $\frac{x^2 + 7x + 12}{x^2 + 2x - 8}$

Multiplying Rational Expressions

Example 2 Find $\dfrac{x^2-16}{2x+8}\cdot\dfrac{x+4}{x^2+8x+16}$.

$$\frac{x^2-16}{2x+8}\cdot\frac{x+4}{x^2+8x+16}=\frac{(x-4)(x+4)}{2(x+4)}\cdot\frac{x+4}{(x+4)(x+4)}\quad\text{Factor.}$$

$$=\frac{(x-4)\cancel{(x+4)}^{1}}{2\cancel{(x+4)}_{1}}\cdot\frac{\cancel{x+4}^{1}}{(x+4)\cancel{(x+4)}_{1}}\quad\text{Simplify.}$$

$$=\frac{x-4}{2x+8}\quad\text{Multiply.}$$

Multiply the rational expressions:

31. $\dfrac{x^2-64}{2x+16}\cdot\dfrac{x+8}{x^2+16x+64}$

32. $\dfrac{x^2+7x+12}{x^2+2x-8}\cdot\dfrac{x^2+3x-12}{x^2+2x-8}$

Multiplying Rational Expressions

Example 2 Find $\dfrac{x^2+6x-27}{x^2+11x+18}\div\dfrac{x-3}{x^2+x-2}$.

$$\frac{x^2+6x-27}{x^2+11x+18}\div\frac{x-3}{x^2+x-2}=\frac{x^2+6x-27}{x^2+11x+18}\times\frac{x^2+x-2}{x-3}$$

$$=\frac{(x+9)(x-3)}{(x+9)(x+2)}\times\frac{(x+2)(x-1)}{x-3}$$

$$=\frac{{}^{1}\cancel{(x+9)}\cancel{(x-3)}^{1}}{{}_{1}\cancel{(x+9)}\cancel{(x+2)}_{1}}\times\frac{{}^{1}\cancel{(x+2)}(x-1)}{\cancel{x-3}_{1}}$$

$$=x-1$$

Divide the rational expressions:

33. $\dfrac{12c^2d}{5a^2b^2}\div\dfrac{c^2d^2}{10ab}$

34. $\dfrac{x^2-9}{2x^2+13x-7}\div\dfrac{x+3}{4x^2-1}$

Adding and Subtracting Rational Expressions

Step 1	If necessary, find equivalent fractions that have the same denominator.
Step 2	Add or subtract the numerators.
Step 3	Combine any like terms in the numerator.
Step 4	Factor if possible.
Step 5	Simplify if possible.

Example Simplify $\frac{6}{2x^2 + 2x - 12} - \frac{2}{x^2 - 4}$.

$\frac{6}{2x^2 + 2x - 12} - \frac{2}{x^2 - 4}$

$= \frac{6}{2(x + 3)(x - 2)} - \frac{2}{(x - 2)(x + 2)}$ Factor the denominators.

$= \frac{6(x + 2)}{2(x + 3)(x - 2)(x + 2)} - \frac{2 \cdot 2(x + 3)}{2(x + 3)(x - 2)(x + 2)}$ The LCD is $2(x + 3)(x - 2)(x + 2)$.

$= \frac{6(x + 2) - 4(x + 3)}{2(x + 3)(x - 2)(x + 2)}$ Subtract the numerators.

$= \frac{6x + 12 - 4x - 12}{2(x + 3)(x - 2)(x + 2)}$ Distributive Property

$= \frac{2x}{2(x + 3)(x - 2)(x + 2)}$ Combine like terms.

$= \frac{x}{(x + 3)(x - 2)(x + 2)}$ Simplify.

35. $\frac{3x+3}{x^2+2x+1} + \frac{x-1}{x^2-1}$

36. $\frac{4}{4x^2-4x+1} - \frac{5x}{20x^2-5}$

Rational Equations

Example Solve $\frac{9}{10} + \frac{2}{x+1} = \frac{2}{5}$.

$\frac{9}{10} + \frac{2}{x+1} = \frac{2}{5}$	Original equation
$10(x+1)\left(\frac{9}{10} + \frac{2}{x+1}\right) = 10(x+1)\left(\frac{2}{5}\right)$	Multiply each side by 10(x + 1).
$9(x+1) + 2(10) = 4(x+1)$	Multiply.
$9x + 9 + 20 = 4x + 4$	Distributive Property
$5x = -25$	Subtract 4x and 29 from each side.
$x = -5$	Divide each side by 5.

Check

$\frac{9}{10} + \frac{2}{x+1} = \frac{2}{5}$	Original equation
$\frac{9}{10} + \frac{2}{-5+1} \stackrel{?}{=} \frac{2}{5}$	$x = -5$
$\frac{18}{20} - \frac{10}{20} \stackrel{?}{=} \frac{2}{5}$	Simplify.
$\frac{2}{5} = \frac{2}{5}$	

Solve each equation:

37. $\frac{2x+1}{3} - \frac{x-5}{4} = \frac{1}{2}$

38. $\frac{3m+2}{5m} + \frac{2m-1}{2m} = 4$

Multiplying Monomials

Example Simplify $(-2ab^2)^3(a^2)^4$.

$(-2ab^2)^3(a^2)^4 = (-2ab^2)^3(a^8)$	Power of a Power
$= (-2)^3(a^3)(b^2)^3(a^8)$	Power of a Product
$= (-2)^3(a^3)(a^8)(b^2)^3$	Group the coefficients and the variables
$= (-2)^3(a^{11})(b^2)^3$	Product of Powers
$= -8a^{11}b^6$	Power of a Power

The product is $-8a^{11}b^6$.

Example 2 Simplify $(-4a^3b)(3a^2b^5)$.

$$(-4a^3b)(3a^2b^5) = (-4)(3)(a^3 \cdot a^2)(b \cdot b^5)$$
$$= -12(a^{3+2})(b^{1+5})$$
$$= -12a^5b^6$$

The product is $-12a^5b^6$.

39. $(-5xy)(4x^2)(-3y^4)$

40. $(2x^3y^2z^5)^3(-3xy^2z)^2$

<u>Dividing Monomials</u>

Example **Simplify $\frac{4a^{-3}b^6}{16a^2b^6c^{-5}}$. Assume that the denominator is not equal to zero.**

$\frac{4a^{-3}b^6}{16a^2b^6c^{-5}} = \left(\frac{4}{16}\right)\left(\frac{a^{-3}}{a^2}\right)\left(\frac{b^6}{b^6}\right)\left(\frac{1}{c^{-5}}\right)$ Group powers with the same base.

$= \frac{1}{4}(a^{-3-2})(b^{6-6})(c^5)$ Quotient of Powers and Negative Exponent Properties

$= \frac{1}{4}a^{-5}b^0c^5$ Simplify.

$= \frac{1}{4}\left(\frac{1}{a^5}\right)(1)c^5$ Negative Exponent and Zero Exponent Properties

$= \frac{c^5}{4a^5}$ Simplify.

The solution is $\frac{c^5}{4a^5}$.

41. $\frac{(3xy)^2z^{-4}}{x^{-1}y^2z^7}$

42. $\frac{(-2mn^2)^{-3}}{4m^{-6}n^4}$

<u>Multiplying Polynomials</u>

Example **Find $(3x + 2)(2x^2 - 4x + 5)$.**

$(3x + 2)(2x^2 - 4x + 5)$

$= 3x(2x^2 - 4x + 5) + 2(2x^2 - 4x + 5)$ Distributive Property

$= 6x^3 - 12x^2 + 15x + 4x^2 - 8x + 10$ Distributive Property

$= 6x^3 - 8x^2 + 7x + 10$ Combine like terms.

The product is $6x^3 - 8x^2 + 7x + 10$.

Multiply the polynomials:

43. $(x - 3)(x^2 - 4x + 2)$

44. $(3x^2 - 2x + 1)(2x^2 - 3x - 4)$

Synthetic Division

Use synthetic division to find $(2x^3 - 5x^2 + 5x - 2) \div (x - 1)$.

Step 1	Write the terms of the dividend so that the degrees of the terms are in descending order. Then write just the coefficients.	$2x^3 - 5x^2 + 5x - 2$ 2 −5 5 −2
Step 2	Write the constant r of the divisor $x - r$ to the left. In this case, $r = 1$. Bring down the first coefficient, 2, as shown.	1⌋ 2 −5 5 −2 2
Step 3	Multiply the first coefficient by r, $1 \cdot 2 = 2$. Write their product under the second coefficient. Then add the product and the second coefficient: $-5 + 2 = -3$.	1⌋ 2 −5 5 −2 2 2 −3
Step 4	Multiply the sum, −3, by r: $-3 \cdot 1 = -3$. Write the product under the next coefficient and add: $5 + (-3) = 2$.	1⌋ 2 −5 5 −2 2 −3 2 −3 2
Step 5	Multiply the sum, 2, by r: $2 \cdot 1 = 2$. Write the product under the next coefficient and add: $-2 + 2 = 0$. The remainder is 0.	1⌋ 2 −5 5 −2 2 −3 2 2 −3 2 0

Thus, $(2x^3 - 5x^2 + 5x - 2) \div (x - 1) = 2x^2 - 3x + 2$.

Use synthetic division to divide:

45. $(3x^3 - 7x^2 + 9x - 14) \div (x - 2)$

46. $(x^4 - 4x^3 + x^2 + 7x - 2) \div (x + 3)$

Identifying Rational Zeros

Example **List all of the possible rational zeros of each function.**

a. $f(x) = 3x^4 - 2x^2 + 6x - 10$

If $\frac{p}{q}$ is a rational root, then p is a factor of -10 and q is a factor of 3. The possible values for p are $\pm1, \pm2, \pm5$, and ±10. The possible values for q are ±1 and ±3. So all of the possible rational zeros are $\frac{p}{q} = \pm1, \pm2, \pm5, \pm10, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{5}{3}$, and $\pm\frac{10}{3}$.

List all of the possible zeros of each function:

47. $f(x) = 6x^7 - 3x^4 + 12x^3 + 18x^2 - 9x + 21$

48. $f(x) = x^8 - 6x^5 - 3x^4 + x^3 + 4x^2 - 120$

Roots and Zeros

Example **Find all of the zeros of $f(x) = x^4 - 15x^2 + 38x - 60$.**

Since $f(x)$ has degree 4, the function has 4 zeros.

$f(x) = x^4 - 15x^2 + 38x - 60$ $f(-x) = x^4 - 15x^2 - 38x - 60$

Since there are 3 sign changes for the coefficients of $f(x)$, the function has 3 or 1 positive real zeros. Since there is 1 sign change for the coefficients of $f(-x)$, the function has 1 negative real zero. Use synthetic substitution to test some possible zeros.

2	1	0	−15	38	−60
		2	4	−22	32
	1	2	−11	16	−28

3	1	0	−15	38	−60
		3	9	−18	60
	1	3	−6	20	0

So 3 is a zero of the polynomial function. Now try synthetic substitution again to find a zero of the depressed polynomial.

−2	1	3	−6	20
		−2	−2	16
	1	1	−8	36

−4	1	3	−6	20
		−4	4	8
	1	−1	−2	28

−5	1	3	−6	20
		−5	10	−20
	1	−2	4	0

So -5 is another zero. Use the Quadratic Formula on the depressed polynomial $x^2 - 2x + 4$ to find the other 2 zeros, $1 \pm i\sqrt{3}$.

The function has two real zeros at 3 and -5 and two imaginary zeros at $1 \pm i\sqrt{3}$.

Find all the zeros of the function:

49. $f(x) = x^3 - 10x^2 + 34x - 40$

50. $f(x) = x^4 - 3x^3 + 21x^2 - 75x - 100$

Logarithmic Functions

Evaluate each expression. Work on a separate sheet of paper. Make sure to show the exponential equation. Leave your answer in simplest fraction form, if necessary.

1. $\log_5 1$
2. $\log_4 \frac{1}{16}$
3. $\log_{\frac{1}{25}} 5$
4. $\log_9 27$
5. $\log_4 128$
6. $\log_{27} \frac{1}{3}$

Solve each equation. Work on a separate sheet of paper. Show all work. Leave your answer in simplest fraction form, if necessary.

7. $\log_9 x = \frac{3}{2}$
8. $\log_4 x = -\frac{3}{2}$
9. $\log_{10} x^2 = -4$
10. $\log_3(x+2) = 5$
11. $\log_6(2x-1) = 3$
12. $\log_4(2x) = -\frac{1}{2}$
13. $\log_8(x-5) = \frac{2}{3}$

Solve each equation. Work on a separate sheet of paper. Show all work. Leave you answer in simplest fraction form, if necessary. Important: When no base is shown, the base is 10.

14. $\log_6(2x-3) = \log_6 12 - \log_6 3$
15. $\log(x+2) - \log x = 2\log 4$
16. $3\log_2 x - 2\log_2(5x) = 2$
17. $2\log_4(x+1) = \log_4(11-x)$
18. $\log x + \log(3x-5) = \log 2$
19. $\log(-4-x) + \log 3 = \log(2-x)$
20. $\log x - \log(x+6) = \frac{1}{2}\log 9$
21. $\log_2(x+7) + \log_2 x = 3$
22. $\log_3(x+3) + \log_3(x+5) = 1$
23. $\log(x+3) = 1 + \log(x-2)$
24. $\log(57x) = 2 + \log(x-2)$
25. $\log_5(x+3) - \log_5(2x-1) = 2$
26. $\log_2(5y+2) - 1 = \log_2(1-2y)$
27. $\log(c^2-1) - 2 = \log(c+1)$
28. $\log_7 x + 2\log_7 x - \log_7 3 = \log_7 72$
29. $\log_{16}(9x+5) - \log_{16}(x^2-1) = \frac{1}{2}$
30. $3\log_5(x^2+9) - 6 = 0$

Next two pages including this page are optional and extra credit on this assignment.

Prerequisite Skills – Exponent Rules, The Quadratic Formula Factoring and Solving Polynomials

Simplify using exponent rules. Assume that no variable equals zero. Write all exponents as POSITIVE.

1. $\dfrac{x^{-2}y}{x^4y^{-1}}$
2. $\dfrac{12m^8y^6}{-9my^4}$
3. $\left(4a^3c^2\right)^3\left(-3ac^4\right)^2$
4. $\left(\dfrac{5a^7}{2b^5c}\right)^3$
5. $\left(\dfrac{7m^{-1}n^3}{m^{-1}n^2}\right)^{-1}$
6. $\dfrac{\left(3x^{-2}y^3\right)\left(5xy^{-8}\right)}{\left(x^3\right)^4\cdot y^{-2}}$

Solve each equation using the quadratic formula.

1. $2x^2-5x+3=0$
2. $2x^2-x-13=2$
3. $2x^2-x-4=2$
4. $8x^2-4x=18$
5. $10x^2+9=x$
6. $x^2=9x-20$
7. $9x^2-11=6x$
8. $4x^2-8=x$

Solve each equation by factoring.

9. $(3n-2)(4n+1)=0$
10. $m^2-3m=0$
11. $(5n-1)(n+1)=0$
12. $(n+2)(2n+5)=0$
13. $3k^2+72=33k$
14. $n^2=-18-9n$
15. $7v^2-42=-35v$
16. $k^2=-4k-4$
17. $-2v^2-v+12=-3v^2+6v$

Solve each polynomial.

18. $f(x)=x^4-4x^3+9x^2-20x+20$
19. $f(x)=4x^4+13x^2+9$
20. $f(x)=x^4+6x^3+11x^2+12x+18$
21. $f(x)=2x^3-7x^2-10x+35$
22. $f(x)=3x^4+5x^3+25x^2+45x-18$
23. $f(x)=x^4-1$
24. $f(x)=x^4+3x^3-19x^2+27x-252$

Domain, Functions and Inverses

Find the domain of each function.

1. $h(x) = 4x - 3$
2. $g(x) = 18 - 5x$
3. $f(x) = \dfrac{2x}{x-3}$
4. $f(x) = \dfrac{x+5}{x+4}$
5. $g(x) = \dfrac{x+3}{x(x+2)}$
6. $h(x) = \dfrac{x-2}{x^2 - 16x + 60}$
7. $g(x) = \dfrac{4}{x^2 - 4}$
8. $f(x) = \sqrt{x-2}$
9. $h(x) = \dfrac{3x}{\sqrt{x-5}}$
10. $f(x) = \dfrac{5}{|x+3|}$
11. $g(x) = \dfrac{x+1}{x^2 + 4x}$
12. $h(x) = \dfrac{x+2}{\sqrt{9 - x^2}}$
13. $j(x) = \dfrac{x}{|x-10|}$
14. $f(x) = \dfrac{\sqrt{8-x}}{x}$

Find $(f+g)(x)$, $(f-g)(x)$, $(f \cdot g)(x)$, and $\left(\dfrac{f}{g}\right)(x)$, given the following:

15. $f(x) = \dfrac{1}{x}$, $g(x) = 7 - x$
16. $f(x) = \dfrac{1}{2-3x}$, $g(x) = \dfrac{2}{3x-2}$
17. $f(x) = \dfrac{3x+5}{2}$, $g(x) = \dfrac{2x-5}{3}$

Find $[f \circ g](x)$ for each $f(x)$ and $g(x)$. Also state the domain of the composition.

18. $f(x) = x^2$ and $g(x) = \dfrac{1}{x^3}$
19. $f(x) = \dfrac{x}{x-2}$ and $g(x) = \dfrac{3}{x}$
20. $f(x) = \dfrac{x-1}{x-2}$ and $g(x) = \dfrac{x-3}{x-4}$
21. $f(x) = x^2 - 16$ and $g(x) = \sqrt{x}$

Inverses: Find the inverse of each function.

22. $f(x) = x^3$
23. $h(x) = \dfrac{1}{x}$
24. $w(x) = 2x + 1$
25. $g(x) = x^2 + 1$, for $x \geq 0$
26. $r(x) = \sqrt[5]{2x+1}$

Showing Inverses by Composition: For each problem, find *f(g(x))* and *g(f(x))*. Then determine whether *f* and *g* are inverses.

27. $f(x) = x^2$, $g(x) = \sqrt{x}$
28. $f(x) = x^3$, $g(x) = \sqrt[3]{x}$
29. $f(x) = \dfrac{1}{x} + 2$, $g(x) = \dfrac{1}{x-2}$
30. $f(x) = 2x + 1$, $g(x) = \dfrac{x}{2} - 1$