AP Calculus AB/BC Summer Assignment 2018

Welcome!

This packet includes a sampling of problems that students entering AP Calculus AB/BC should be able to answer. The questions are organized by topic:

A – Basic Algebra Skills	T – Trigonometry
S – Solving	L – Logarithms and Exponential Functions
F – Functions	R – Rational Expressions and Equations
G – Graphing	PO – Polar Functions
PA – Parametric Functions	PF – Partial Functions
G – Graphing Functions	M – More Basic Algebra Review

Students entering AP Calculus AB/BC absolutely must have a strong foundation in algebra and trigonometry (Pre-Calculus). Most questions in this packet were included because they concern skills and concepts that will be used extensively in AP Calculus AB/BC. Others have been included not so much because they are skills that are used frequently, but because being able to answer them indicates a strong grasp of important mathematical concepts and—more importantly—the ability to problem-solve.

It is extremely important for all students to review the concepts contained in this packet and to be prepared for an assessment of prerequisite skills to take place within the first 2-3 days of school.

Students whose scores show they were not prepared for the assessment probably either (a) don't have the mathematical prerequisite skills necessary for success in AP Calculus AB/BC , or (b) don't have the work ethic necessary for success in AP Calculus. You are expected to approach problems with the mathematical toolkit needed to do the calculations and the mathematical understanding needed to make sense of unusual problems.

Now, you are encouraged to take a deep breath and start working. If you have the basics down and you put in the work needed, you'll see how amazing AP Calculus is! AP Calculus is very challenging, demanding, rewarding, and—to put it simply—totally awesome. Please let me know if you have any questions at sduzyol@fultonscienceacademy.org.

A: Basic Algebra Skills

A1. True or false. If false, change what is underlined to make the statement true.

a. $\left(x^3\right)^4 = \underline{x^{12}}$ T F

b. $x^{\frac{1}{2}}x^3 = \underline{x^{\frac{3}{2}}}$ T F

c. $(x+3)^2 = \underline{x^2+9}$ T F

d. $\dfrac{x^2-1}{x-1} = \underline{x}$ T F

e. $(4x+12)^2 = \underline{16}(x+3)^2$ T F

f. $\underline{3} + 2\sqrt{x-3} = 5\sqrt{x-3}$ T F

g. If $(x+3)(x-10) = \underline{2}$, then $x+3 = \underline{2}$ or $x-10 = \underline{2}$. T F

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T: Trigonometry

You should be able to answer these quickly, *without* using calculator and without referring to (or drawing) a unit circle.

T1. Evaluate Trig Functions without a calculator:

1. $\cos\pi$
2. $\sin\dfrac{\pi}{6}$
3. $\sec 210^\circ$
4. $\tan 90^\circ$
5. $\csc(-150)$
6. $\csc\dfrac{3\pi}{2}$
7. $\cos 0$
8. $\sin^{-1}\dfrac{-1}{2}$
9. $\operatorname{Cos}^{-1}\left(\dfrac{-\sqrt{3}}{2}\right)$
10. $\tan^{-1} 1$
11. $\arcsin 0$
12. $\operatorname{Tan}^{-1}\left(-\sqrt{3}\right)$
13. $\sin\dfrac{2\pi}{3}$
14. $\operatorname{Sin}^{-1}\left(\dfrac{\sqrt{2}}{2}\right)$
15. $\arctan 0$

T2. Find the value of each expression, in exact form.

a. $\sin\frac{2\pi}{3}$

b. $\cos\frac{11\pi}{6}$

c. $\tan\frac{3\pi}{4}$

d. $\sec\frac{5\pi}{3}$

e. $\csc\frac{7\pi}{4}$

f. $\cot\frac{5\pi}{6}$

<u>Note</u>: You will need to know your trig identities, Sum & Difference & Double Angle Formulas:

Memorize the following Trig Identities:

$\sin^2\theta + \cos^2\theta = 1$ $\sec^2\theta = 1 + \tan^2\theta$ $\csc^2\theta = 1 + \cot^2\theta$

$\sin(-\theta) = -\sin\theta$ $\cos(-\theta) = \cos\theta$ $\tan(-\theta) = -\tan\theta$

$\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$ $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$

$\sin 2\theta = 2\sin\theta\cos\theta$ $\cos 2\theta = \cos^2\theta - \sin^2\theta$

T3 Find the value(s) of x in $[0, 2\pi)$ which solve each equation.

a. $\sin x = \frac{\sqrt{3}}{2}$ **b.** $\cos x = -1$

c. $\tan x = \sqrt{3}$ **d.** $\sec x = -2$

e. $\csc x$ is undefined **f.** $\cot x = 1$

T4. Solve the equation. Give *all* real solutions, if any.

a. $\sin 3x = 1$ **b.** $2\sqrt{3}\cos(\pi x) = 3$

c. $\tan 2x = 0$ **d.** $4\sec x + 1 = 9$

e. $\csc(4x+3) = 0$ **f.** $3\cot 6x + \sqrt{3} = 0$

T5. Solve by factoring. Give *all* real solutions, if any.

a. $4\sin^2 x + 4\sin x + 1 = 0$ **b.** $\cos^2 x - \cos x = 0$

c. $\sin x\cos x - \sin^2 x = 0$ **d.** $x\tan x + 3\tan x = x + 3$

T6. Graph each function, identifying x- and y-intercepts, if any, and asymptotes, if any.

a. $y = -\sin(2x)$ **b.** $y = 4 + \cos x$

c. $y = \tan x - 1$ **d.** $y = \sec x + 1$

e. $y = \csc(\pi x)$ **f.** $y = 2\cot x$

S: Solving

S1. Solve by factoring.

a. $x^3 + 5x^2 - x - 5 = 0$

b. $4x^4 + 36 = 40x^2$

c. $(x^3 - 6)^2 + 3(x^3 - 6) - 10 = 0$

d. $x^5 + 8 = x^3 + 8x^2$

S2. Solve by factoring. You should be able to solve each of these *without* multiplying the whole thing out. (In fact, for goodness' sake, please *don't* multiply it all out!)

a. $(x + 2)^2 (x + 6)^3 + (x + 2)(x + 6)^4 = 0$

b. $(2x - 3)^3 (x^2 - 9)^2 + (2x - 3)^5 (x^2 - 9) = 0$

c. $(3x + 11)^5 (x + 5)^2 (2x - 1)^3 + (3x + 11)^4 (x + 5)^4 (2x - 1)^3 = 0$

d. $6x^2 - 5x - 4 = (2x + 1)^2 (3x - 4)^2$

S3. Solve. (*Hint*: Each question *can* be solved by factoring, but there are other methods, too)

a. $a(3a + 2)^{1/2} + 2(3a + 2)^{3/2} = 0$

b. $\sqrt{2x^2 + x - 6} + \sqrt{2x - 3} = 0$

c. $2\sqrt{x + 3} = x + 3$

d. $\dfrac{6}{(2x + 1)^2} + \dfrac{3}{2x + 1} = 1 + \dfrac{2}{2x + 1}$

S4. Solving Inequalities: ***Solve and graph the solution***

a. $|x - 3| > 12$

b. $|x - 3| \le 4$

c. $|10x + 8| > 2$

d. $x^2 - 16 < 0$

e. $x^2 + 6x - 16 \le 0$

f. $x^2 - 3x \ge 10$

L: Logarithms and Exponential Functions

L1. Evaluate Logarithms and Exponentials without a calculator

a. $\log_4 64$ **b.** $\log_3 \frac{1}{9}$ **c.** $\log 10$ **d.** $\ln e$

e. $\ln 1$ **f.** $\ln e^3$ **g.** $3^{\log_3 7}$ **h.** $4^{\log_4 \sin x}$

L2. Expand as much as possible.

a. $\ln x^2 y^3$ **b.** $\ln \frac{x+3}{4y}$

c. $\ln 3\sqrt{x}$ **d.** $\ln 4xy$

L3. Condense into the logarithm of a single expression.

a. $4\ln x + 5\ln y$ **b.** $\frac{2}{3}\ln a + 5\ln 2$

c. $\ln x - \ln 2$ **d.** $\frac{\ln x}{\ln 2}$

(contrast with part **c**)

L4. Solve. Give your answer in exact form *and* rounded to three decimal places.

a. $\ln(x+3) = 2$ **b.** $\ln x + \ln 4 = 1$

c. $\ln x + \ln(x+2) = \ln 3$ **d.** $\ln(x+1) - \ln(2x-3) = \ln 2$

L5. Solve. Give your answer in exact form *and* rounded to three decimal places.

a. $e^{4x+5} = 1$ **b.** $2^x = 8^{4x-1}$

c. $100e^{x\ln 4} = 50$ **d.** $2^x = 3^{x-1}$

(need rounded answer only in **d**)

L6. Round final answers to 3 decimal places.

a. At $t = 0$ there were 140 million bacteria cells in a petri dish. After 6 hours, there were 320 million cells. If the population grew exponentially for $t \geq 0$...

...how many cells were in the dish 11 hours after the experiment began?

...after how many hours will there be 1 billion cells?

b. The *half-life* of a substance is the time it takes for half of the substance to decay. The *half-life* of Carbon-14 is 5568 years. If the decay is exponential...

...what percentage of a Carbon-14 specimen decays in 100 years?

...how many years does it take for 90% of a Carbon-14 specimen to decay?

F: FUNCTIONS

Graph each of the following Parent Functions and be familiar with these graphs

1. $f(x) = x$	2. $f(x) = x^2$	3. $f(x) = x^3$
4. $f(x) = \|x\|$	5. $f(x) = \sqrt{x}$	6. $f(x) = \frac{1}{x}$
7. $f(x) = \frac{1}{x^2}$	8. $f(x) = e^x$	9. $f(x) = \ln x$
10. $f(x) = \sin x$	11. $f(x) = \cos x$	12. $f(x) = \tan x$
13. $f(x) = \tan^{-1} x$	14. $f(x) = x^{\frac{2}{3}}$	15. $f(x) = \frac{1}{1+x^2}$
16. $f(x) = [x]$	17. $f(x) = \sqrt{1 - x^2}$	18. $f(x) = \frac{\|x\|}{x}$

Analyzing Functions

F1. Increasing/Decreasing

Determine the interval(s) over which f(x) is:

a. Increasing ____________________

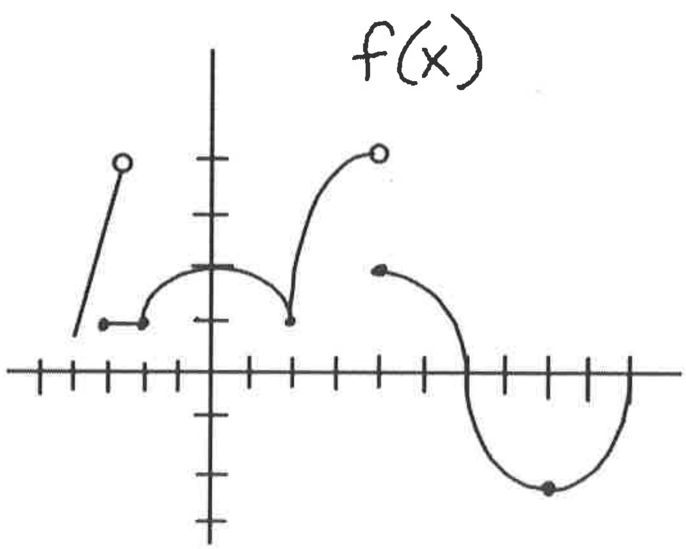
b. Decreasing ____________________

c. Constant ____________________

d. Linear ____________________

e. Concave Up ____________________

f. What are the zeros of f? ____________



g. For what values of x is f(x) discontinuous? ________________

F2. Compositions

1. Let $f(x) = 3x^2$ and $g(x) = \frac{x-9}{x+1}$, find the following:

a. $f(g(x))$ b. $g(f(x))$ c. $f^{-1}(x)$

d. Domain, Range, and Zeros of f(x) e. Domain, Range, and Zeros of g(x)

Find f^{-1} and verify that $(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$.

2. $f(x) = 2x + 3$ 3. $f(x) = x^3 - 1$

F3. Piecewise Functions:

Graph and then evaluate the function at the indicated points.

1. $f(x)=\begin{cases} 3x+2, & x>3 \\ -x+4, & x\leq 3 \end{cases}$

a. f(2) b. f(3) c. f(5)

2. $f(x)=\begin{cases} x^2\text{-}1, & x<-2 \\ 4, & -2\leq x\leq 1 \\ 3x+1, & 1<x<3 \\ x^2-1, & x>3 \end{cases}$

a. f(-3) b. f(-2) c. f(2) d. f(5) e) f(3)

F4. Even/Odd Functions

Show work to determine if the relation is even, odd, or neither.

a. $f(x)=2x^2-7$ **b.** $f(x)=-4x^3-2x$ **c.** $f(x)=4x^2-4x+4$

d. $f(x)=x-\dfrac{1}{x}$ **e.** $f(x)=|x|-x^2+1$ **f.** $f(x)=\sin x+x$

F5. Domains of Functions: Find the Domain of each.

a. $y=\dfrac{3x-2}{4x+1}$ **b.** $y=\dfrac{x^2-4}{2x+4}$ **c.** $y=\dfrac{x^2-5x-6}{x^2-3x-18}$

d. $y=\dfrac{2^{2-x}}{x}$ **e.** $y=\sqrt{x-3}-\sqrt{x+3}$ **f.** $y=\dfrac{\sqrt{2x-9}}{2x+9}$

F6. Asymptotes

Find the equation of both Horizontal and Vertical Asymptotes for the following functions. Find the coordinates of any holes.

a. $y=\dfrac{x}{x-3}$ **b.** $y=\dfrac{x+4}{x^2-1}$

c. $y=\dfrac{x^2-2x+1}{x^2-3x-4}$ **d.** $y=\dfrac{x^2-9}{x^3-3x^2-18x}$

R: Rational Expressions and Equations

R1.

	Function	Domain	Hole(s): (x, y) if any	Horiz. Asym., if any	Vert. Asym.(s), if any
a.	$f(x)=\dfrac{4x^2+7x-15}{8x^2-14x+5}$				
b.	$f(x)=\dfrac{3(4+x)^2-48}{x}$				
c.	$f(x)=\dfrac{6x+4}{\sqrt{3x^2-10x-8}}$		skip	skip	

R2. Write the equation of a function that has...

a. asymptotes $y=4$ and $x=1$, and a hole at (3, 5)

b. holes at (-2, 1) and (2, -1), an asymptote $x=0$, and no horizontal asymptote

R3. Find the x-coordinates where the function's output is zero and where it is undefined.

a. For what real value(s) of x, if any, is the output of the function $f(x)=\dfrac{x^2+4}{e^{6x}-1}$

...equal to zero? ...undefined?

b. For what real value(s) of x, if any, is the output of $g(x)=\dfrac{\cos^2(\pi x)}{\sin x+2}$...

...equal to zero? ...undefined?

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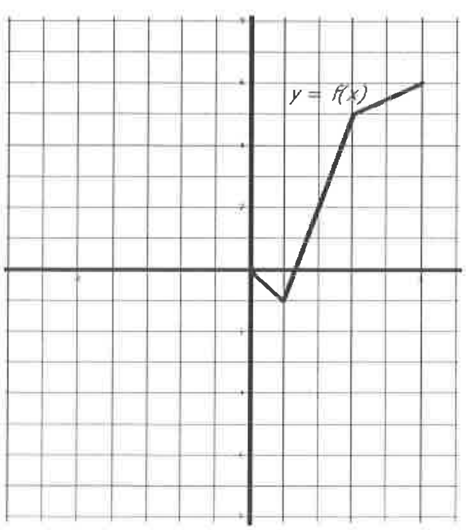
G: Graphing

G1. PART of the graph of f is given. Each gridline represents 1 unit.



a. Complete the graph to make f an EVEN function.

b. What are the domain and range of f_{even}?

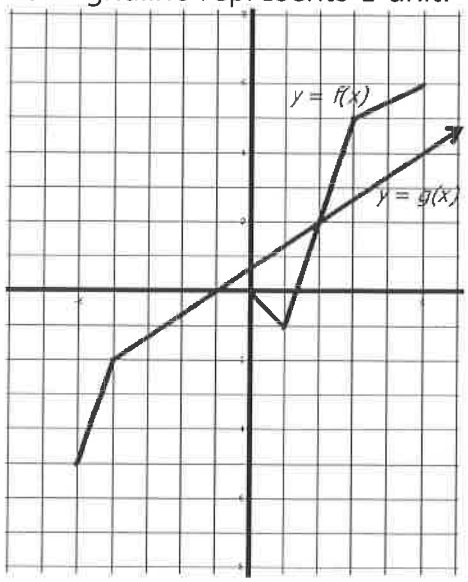
c. What is $f_{even}(-3)$?

d. Complete the graph to make f an ODD function.

e. What are the domain and range of f_{odd}?

f. What is $f_{odd}(-3)$?

G2. The graphs of f and g are given. Answer each question, if possible. If impossible, explain why. Each gridline represents 1 unit.



a. $f^{-1}(5) =$

b. $f(g(5)) =$

c. $(g \circ f)(3) =$

d. Solve for x: $f(g(x)) = 5$

e. Solve for x: $f(x) = g(x)$

For parts **f** – **i**, respond in interval notation.

f. For what values of x is $f(x)$ increasing?

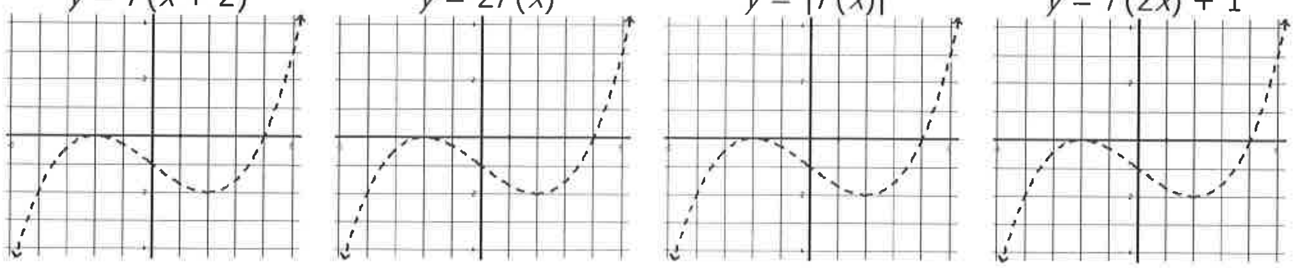
g. For what values of x is $g(x)$ positive?

h. Solve for x: $f(x) < 4$

i. Solve for x: $f(x) \geq g(x)$

G3. Given the graph of $y = f(x)$ (dashed graph), sketch each transformed graph.

a. $y = f(x+2)$ **b.** $y = 2f(x)$ **c.** $y = |f(x)|$ **d.** $y = f(2x) + 1$



PO: Polar Functions

PO1. Plot the point $\left(3, -\frac{3\pi}{4}\right)$ and find three additional represents of this point using $-2\pi < \theta < 2\pi$.

PO2. Convert the given points in polar into rectangular coordinates

(a) $\left(\sqrt{3}, \frac{\pi}{6}\right)$, (b) $\left(2, \frac{2\pi}{3}\right)$, (c) $\left(-3, \frac{-3\pi}{4}\right)$ (d) $\left(-2, \frac{5\pi}{6}\right)$.

PO3. Convert the given points in rectangular into polar coordinates,

(a) (0, 2), (b) $\left(-1, \sqrt{3}\right)$, (c) $\left(-\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$ (d) $\left(\sqrt{3}, -1\right)$.

PO4. Convert the following polar equations into rectangular form:

a. r=2 **b.** $\theta = \frac{\pi}{3}$ **c.** $r = \sec\theta$ **d.** $r = 3\cos\theta + 2\sin\theta$

PO5. Convert the following rectangular equations into polar form

a. $y = x$ **b.** $x = 10$ **c.** $x^2 + y^2 = 4$ **d.** $x^2 - y^2 = 4x$

PO6. Sketch the graph of the following polar equation:

$$r = 3 + 2\cos\theta$$

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PA: Parametric Functions

Obtain the rectangular equation by eliminating the parameter. Sketch a graph using the parametric equations:

(a) $x = 2t - 5$, $y = 4t - 7$ (b) $x = 4 - \sqrt{t}$, $y = \sqrt{t}$ (c) $x = t^2$, $y = \sqrt{4 - t^2}$

(d) $x = 4\cos\theta$, $y = 2\sin\theta$ (e) $x = 9\sin^2\theta$, $y = 9\cos^2\theta$ (f) $x = \sec^2\theta - 1$, $y = \tan\theta$

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PF: Partial Fractions

PF1. Find the partial fraction decomposition of

(a) $\dfrac{2x-1}{(x-2)(x-3)}$ (b) $\dfrac{x+7}{x^2 - x - 6}$ (c) $\dfrac{x^2+2}{(x-1)(x+2)(x-3)}$

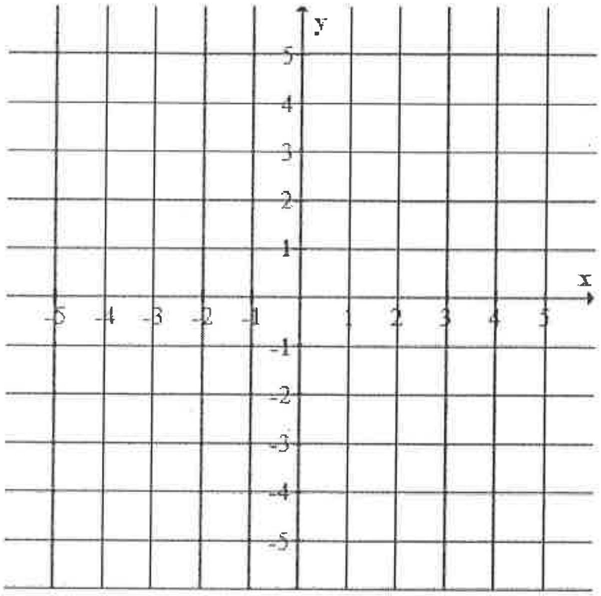
G – Graphing Functions

$y = 1 + \frac{1}{x}$

a) ____________________

b) ____________________

c)



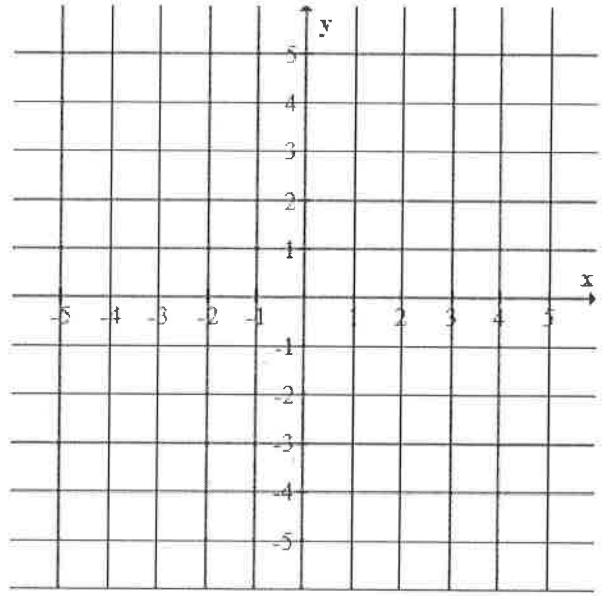
d) ____________________

$y = x^{2 3}$

a) ____________________

b) ____________________

c)



d) ____________________

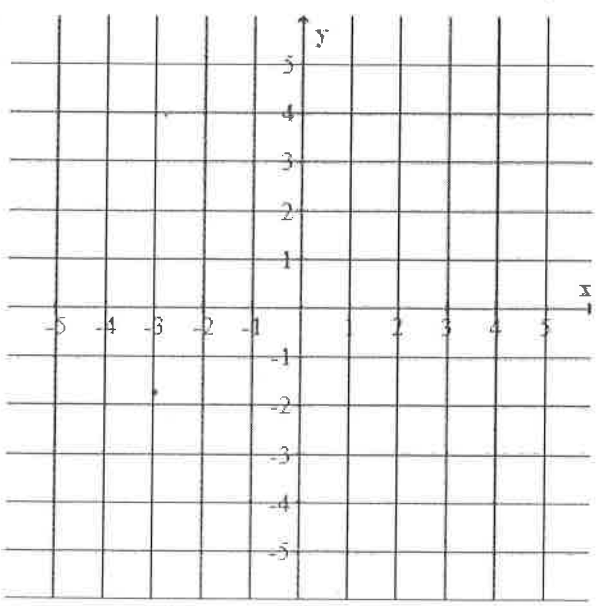
For the following functions, find the a) domain, b) range, c) graph, and d) any symmetries.

$y = -2^x + 3$

a) ____________________

b) ____________________

c)



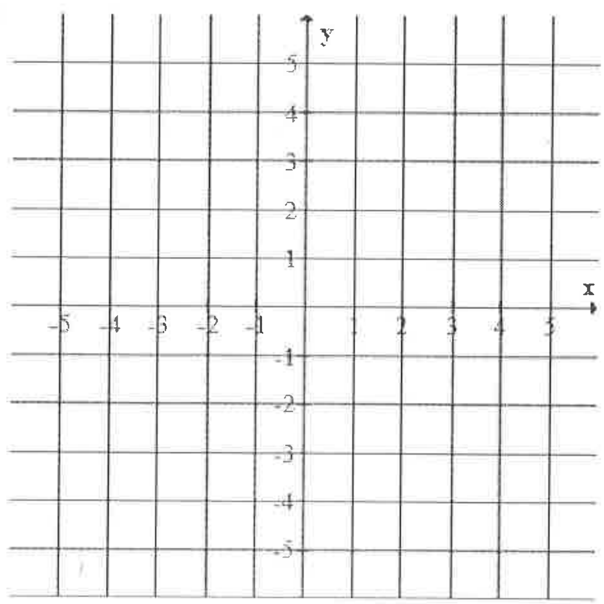
d) ____________________

$y = e^x + 3$

a) ____________________

b) ____________________

c)



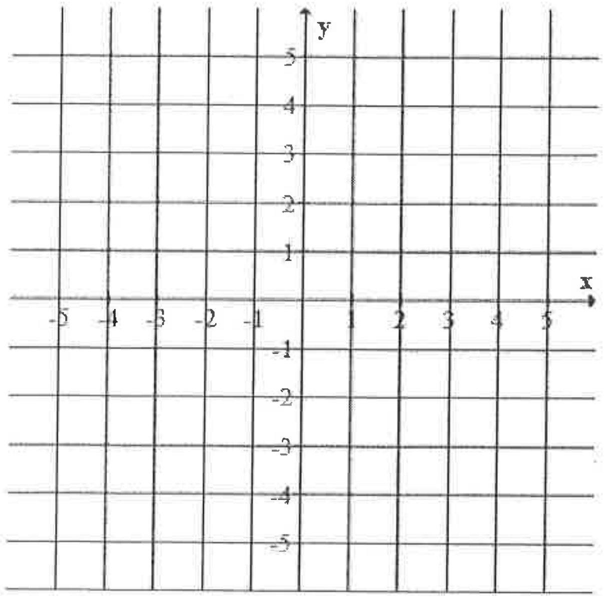
d) ____________________

$y = 3e^{-x} - 2$

a) ____________________

b) ____________________

c)



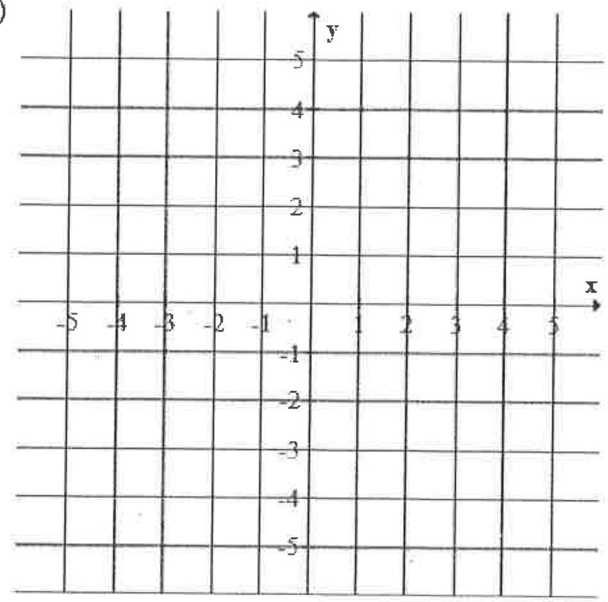
d) ____________________

$y = -2^{-x} - 1$

a) ____________________

b) ____________________

c)



d) ____________________

M – More Basic Algebra Review

1. Solve $xy + 2x + 1 = y \ for \ y$

2. Factor: $x^2(x-1) - 4(x-1)$

3. Solve $\ln(y-1) - ln = x + lnx \ for \ y$

4. Factor $3x^{\frac{3}{2}} - 9x^{\frac{1}{2}} + 6x^{-\frac{1}{2}}$

Simplify each expression.

5. $\dfrac{(x^2)^3 x}{x^7}$

6. $\sqrt{x} \cdot \sqrt[3]{x} \cdot x^{\frac{1}{6}}$

7. $\dfrac{5(x+h)^2 - 5x^2}{h}$

8. $\dfrac{\frac{1}{x} + \frac{4}{x^2}}{3 - \frac{1}{x}}$

Simplify, by factoring first. Leave answers in factored form.

Example:

$$\frac{(x+1)^3(4x-9)-(16x+9)(x+1)^2}{(x-6)(x+1)} = \frac{(x+1)^2[(x+1)(4x-9)-(16x+9)]}{(x-6)(x+1)}$$

$$= \frac{(x+1)^2[4x^2-5x-9-16x-9]}{(x-6)(x+1)}$$

$$= \frac{(x+1)^2[4x^2-21x-18]}{(x-6)(x+1)}$$

$$= \frac{(x+1)^2[(4x+3)(x-6)]}{(x-6)(x+1)}$$

$$= (x+1)(4x+3)$$

9. $(x-1)^3(2x-3)-(2x+12)(x-1)^2$

10. $\frac{(x-1)^2(3x-1)-2(x-1)}{(x-1)^4}$

Simplify by rationalizing the numerator.

Example:

$$\frac{\sqrt{x+4}-2}{x} = \frac{\sqrt{x+4}-2}{x} \bullet \frac{\sqrt{x+4}+2}{\sqrt{x+4}+2} = \frac{x+4-4}{x\left(\sqrt{x+4}+2\right)} = \frac{x}{x\left(\sqrt{x+4}+2\right)} = \frac{1}{\sqrt{x+4}+2}$$

11. $\frac{\sqrt{x+9}-3}{x}$

12. $\frac{\sqrt{x+h}-\sqrt{x}}{h}$

Solve each equation or inequality for x over the set of real numbers.

13. $2x^4 + 3x^4 - 2x^2 = 0$

14. $\frac{2x-7}{x+1} = \frac{2x}{x+4}$

15. $\sqrt{x^2 - 9} = x - 1$

16. $|2x - 3| = 14$

17. $x^2 - 2x - 8 < 0$ (answer in interval notation)

18. $\frac{3x+5}{(x-1)(x^4+7)} = 0$